

Equation of the line & unusual equations of the line

Think of a function as being a bit like a Christian name which is used to distinguish one person from another but we can get a bit of information about the person. For example Ciara would tell us the person is a girl. We can also use a function to distinguish one line (or curve) from another but unlike a Christian name in which often the only information you can get from it is gender a function will actually *give information about the properties and characteristics of the line or curve* it represents.

1) Equation of the line

All lines are represented by the general equation $Y = MX + C$.

$$\text{e.g. } Y = \frac{1}{2}X + 5$$

Here the M represents the slope or the steepness of the line (the larger M is the steeper the line. If M is a negative value then it means the line has a ‘downhill’ slope. If $M = 0$ then it is horizontal). In our example the line has a slope of $\frac{1}{2}$ which means it has a slight uphill slope.

The C represents the Y coordinate of the point where the line crosses the Y axis (i.e. it represents the Y intercept). Every point on the y axis always has an X coordinate of 0 so in our example above the point on the y axis that our example crosses is at the point (0, 5).

Although some lines might have the same slope and some lines might have the same value for C no two lines will have exactly the same slope and value for C at the same time – if they did then they are actually the same line

Note that to correctly identify the slope and C there must be nothing beside the Y just like it’s written in the equation of the line (you can have $1Y$ or $+Y$ which is the same as Y but nothing else). For example in the line

$$-Y = 3X - 2$$

the slope is not $M = 3$ and C is not $C = -2$ because there is a minus in front of Y.

Adjusting the equation by dividing every term by -1 gives $Y = -3X + 2$ so our slope is $M = -3$ and $C = 2$

Note 2 horizontal lines have a slope $M = 0$ & vertical lines have a slope $M = \infty$ or ‘undefined’

2) Unusual equations of the line

Note that sometimes you get equations of a line that at first don't look like an equation of a line. We can show they actually are by making them look like

$$Y = MX + C.$$

For example the equation

$$Y = 5X$$

is a line. When we compare this to the general equation of the line we see that the constant C term is missing. We can make this look like $Y = MX + C$ without changing the overall value of the right hand side of the equation by including '0' for C the constant term,

$$\text{i.e. } Y = 5X + 0$$

because adding the '0' to it doesn't change anything - when you add (or subtract) '0' to some number or term you get that term i.e. $5X + 0 = 5X$. So $Y = 5X$ is an equation of the line. Not only that but our $Y = 5X + 0$ indicates that the line crosses the y axis at 0, or more precisely (0,0)

Other equations of a line which at first don't look like $Y = MX + C$ are equations like

$$X = 2$$

$$\text{or } Y = 3$$

i.e. where one of the letters is equal to a number. What's happening here is some part of these equations is missing and is equal to '0'.

With the $Y = 3$ example above we can make it look like the equation of a line. Here we see the 'X' term is missing. We can include an X term without changing the overall value of the right hand side by including '0X' to give

$$Y = (0)X + 3$$

Since any number multiplied by '0' is equal to '0' then $Y = (0)X + 3$ is the same as $Y = 0 + 3$ which is the same as $Y = 3$.

So $Y = 3$ is another equation of the line. Note that the '0X' does not mean the X is equal to 0. It means *the number in front of X* is 0. If we look closer at the $Y = (0)X + 3$ this is telling us that $Y = 3$ is a line with slope equal to 0 and it crosses Y axis at 3 (or more precisely (0,3)).

We can make $X = 2$ look like an equation of a line by moving the 'X' around to the same side of the equals sign as the constant '2' which will give

$$0 = -X + 2$$

Now we can see it is just the 'Y' term that is missing. We can include a Y term by putting '0Y' on the left side to give

$$0Y = X - 2$$

(again this means the Y isn't equal to 0 but *the number in front of Y* is 0)

If we look closer we see this has the same structure as $Y = MX + C$